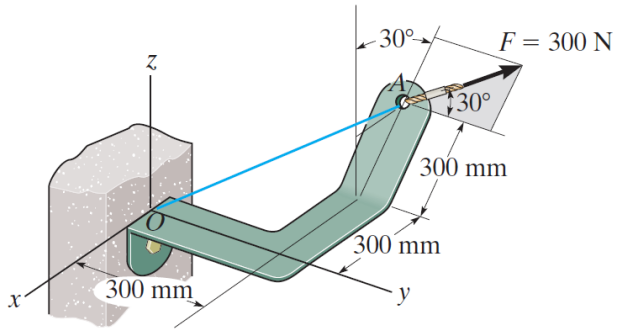
Problem 2-137

Determine the magnitude of the projected component of the force $F = 300$ N acting along line OA.



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Solution

Write the position vectors to the points O and A.

$$\mathbf{r}_O = \langle 0, 0, 0 \rangle \text{ mm}$$

$$\mathbf{r}_A = \langle -300 - 300\sin 30°, 300, 300\cos 30° \rangle \text{ mm}$$

The unit vector going from O to A is

$$\hat{\mathbf{u}}_{OA} = \frac{\mathbf{r}_A - \mathbf{r}_O}{|\mathbf{r}_A - \mathbf{r}_O|} = \frac{\langle -300 - 300\sin 30°, 300, 300\cos 30° \rangle}{\sqrt{(-300 - 300\sin 30°)^2 + (300)^2 + (300\cos 30°)^2}}.$$

Write the force in component form.

$$\mathbf{F} = 300 \langle -\sin 30° \sin 30°, \cos 30°, \sin 30° \cos 30° \rangle \text{ N}$$

Take the dot product of $\mathbf{F}$ with $\hat{\mathbf{u}}_{OA}$ to get the component of the force along line OA.

$$\mathbf{F} \cdot \hat{\mathbf{u}}_{OA} = 300 \langle -\sin 30° \sin 30°, \cos 30°, \sin 30° \cos 30° \rangle \cdot \frac{\langle -300 - 300\sin 30°, 300, 300\cos 30° \rangle}{\sqrt{(-300 - 300\sin 30°)^2 + (300)^2 + (300\cos 30°)^2}} \text{ N}$$

$$= \frac{1}{2}(225 + 150\sqrt{3}) \text{ N}$$

$$\approx 242 \text{ N}$$

Therefore, the magnitude of the force's component along OA is

$$|\mathbf{F} \cdot \hat{\mathbf{u}}_{OA}| = \frac{1}{2}(225 + 150\sqrt{3}) \text{ N} \approx 242 \text{ N}.$$